St Barnabas Church of England Primary School

Barnabas the Encourager

Maths No Problem Calculation Policy 2021-22

Using the Maths No Problem (MNP) textbooks and workbooks

'Our vision is to ignite curiosity and delight in learning so we are ready for an ever changing,challenging world.

We will build each other up to be unique individuals in a diverse community - showing resilience andworking positively together to make every day count.'

Our core values of creativity, courage and compassion underpin our vision.

'So speak encouraging words to one another. Build up hope so you'll all be together in this, no one left out, no one left behind. **1 Thessalonians 5:11**

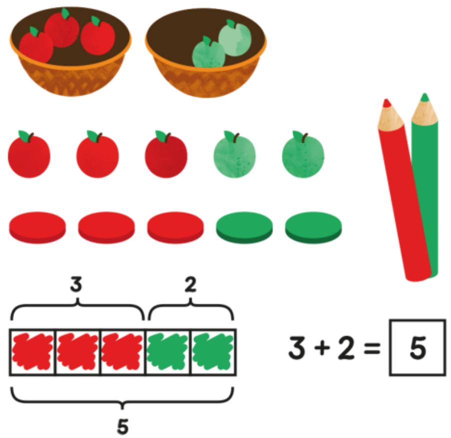
The link with Saint Barnabas the Encourager is at the heart of our vision(Acts of the Apostles)

Concrete Pictorial Abstract approach

One of the key learning principles behind the Singapore maths textbooks is the concrete pictorial abstract approach, often referred to as the **Concrete – Pictorial-Abstract (CPA)** approach.

The concrete-pictorial-abstract approach, based on research by psychologist Jerome Bruner, suggests that there are three steps (or representations) necessary for pupils to develop understanding of a concept. Reinforcement is achieved by going back and forth between these representations.

Concrete



Pictorial and Abstract

Concrete: children use resources (manipulatives), such as base 10 (dienes); numicon; place value counters; paper to represent a problem. They use these concrete resources to solve the problems.

As they become more confident they may move to **Pictorial** and then **Abstract** representations.

Children will use resources at any age and stage in their mathematical learning. In many lessons the children can move through one or more stage.

Concrete representation

The active stage - a student is first introduced to an idea or a skill by acting it out with real objects. In division, for example, this might be done by separating apples into groups of red ones and green ones or by sharing 12 biscuits amongst 6 children. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation

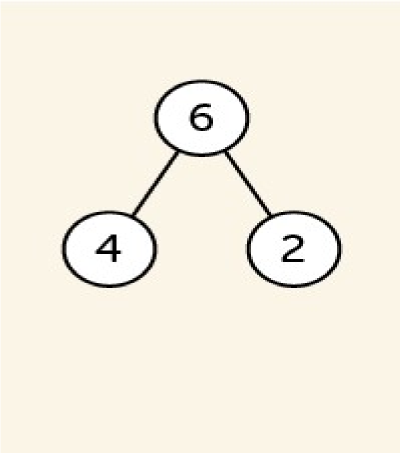
The iconic stage - a student has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem. In the case of a division exercise this could be the action of circling objects.

Abstract representation

The symbolic stage - a student is now capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$ this is the ultimate mode, for it is clearly the most mysterious of the three.

Children will build their competence with the methods in the calculation policy as they move through school, using formal written methods by the time they are in Year 6. Children may move backwards and forwards between methods and strategies when faced with new maths content or as they become more proficient in certain areas. For more detail on the expectation for children in different year groups, please refer to the skills progression for each area of maths.

Number Bonds

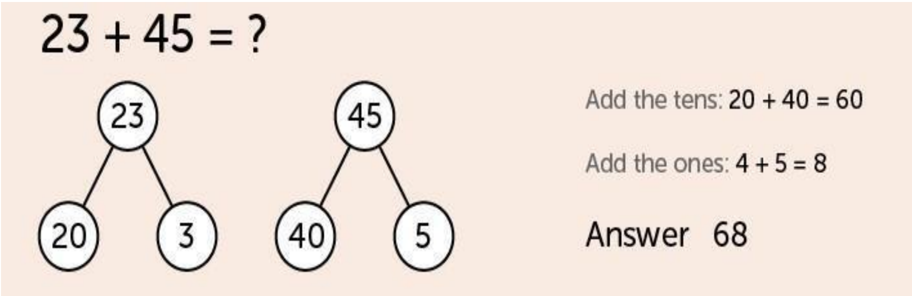


Number bonds are usually symbolised in this fashion within Singapore Maths textbooks

In Singapore mathematics number bonds refer to how numbers can be combined or split up, the 'part-part-whole' relationship of numbers.

When talking about number bonds in Singapore maths we are referring to how numbers join together and how they can be split up. A lot of emphasis is put into number bonds from the early year foundation stages so that children can build up their number sense prior to learning addition and subtraction. In the early stages students would be introduced to number bonds with concrete experiences, for example children could be given 6 linking cubes and guided to understand that 2 and 4 make 6, but that 1 and 5 also make 6.

The mastery of number bonds is an important foundation required in subsequent mathematical learning and as a basis in the development of mental strategies. A strong number sense allows students to decide what action to take when trying to solve problems in their head.



An example of how a student would use number sense gained from number bonds to perform a mental calculation

Source: Good practice in primary mathematics: evidence from 20 successful schools, November 2011, 110140

An example of a Maths No Problem textbook and workbook focussing on making number bonds:

Lesson 1

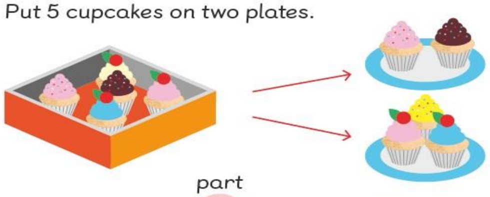
Making Number Bonds

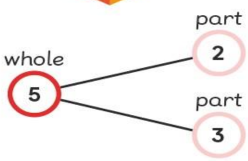
In Focus

How many cupcakes are there on each plate?
Is there another way to put the cupcakes on the two plates?

Let's Learn

1 Put 5 cupcakes on two plates.





2 and 3 make 5.

This is a **number bond**.

Chapter 2

Number Bonds

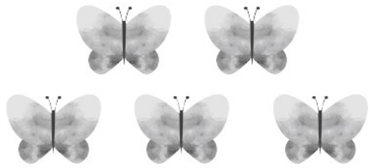
Name: ______________ Class: _____ Date: __________

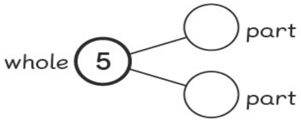
Worksheet 1

Making Number Bonds

1 Complete the number bonds.
Fill in the blanks.

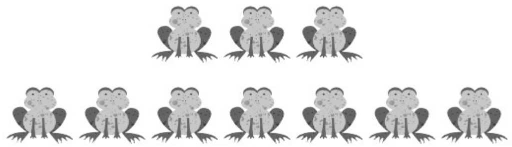
(a)

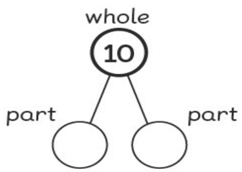




☐ and ☐ make 5.

(b)

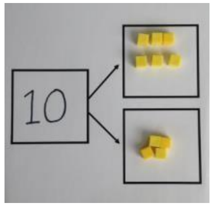
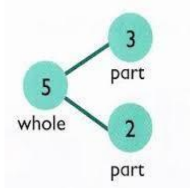
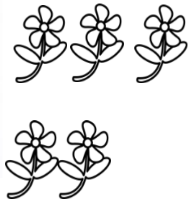
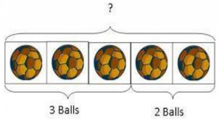
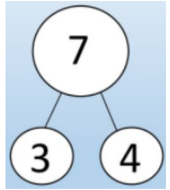
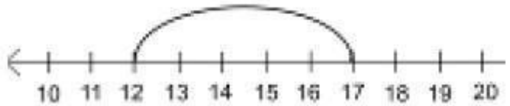


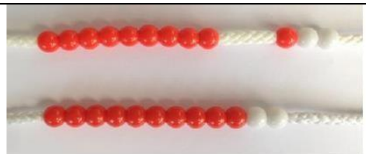
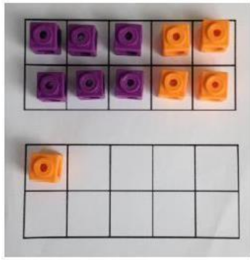
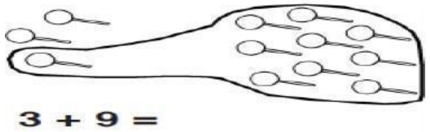
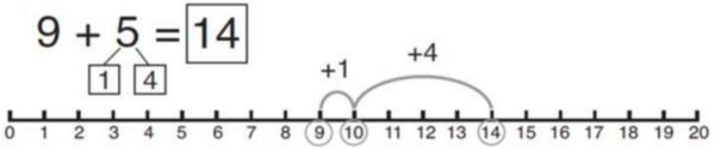
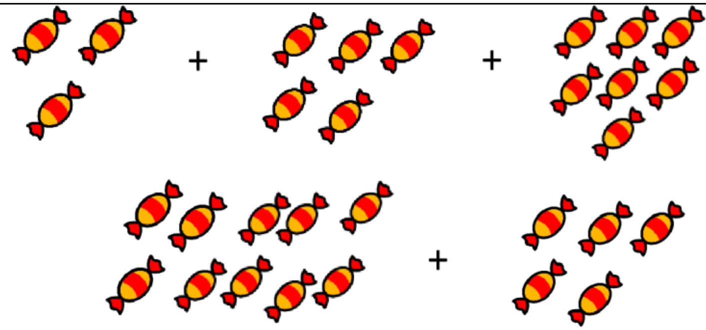
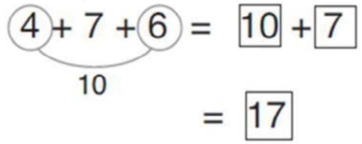


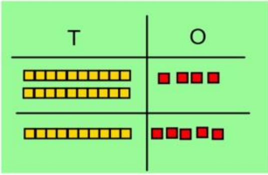
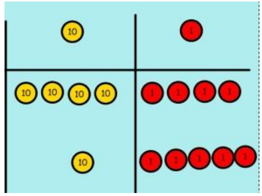
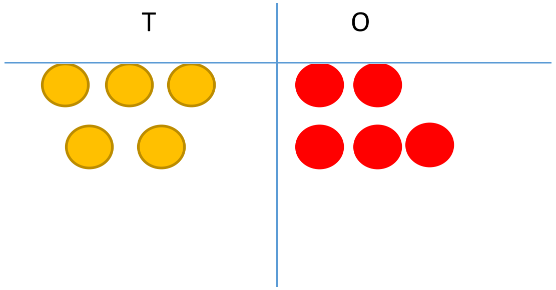
☐ and ☐ make 10.

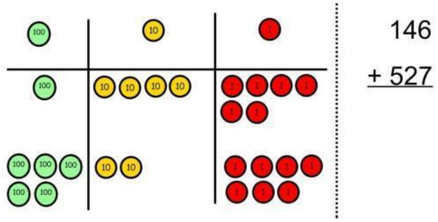
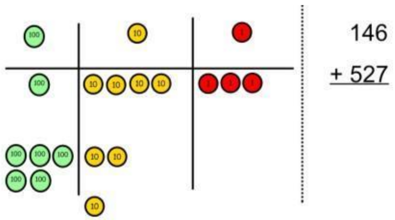
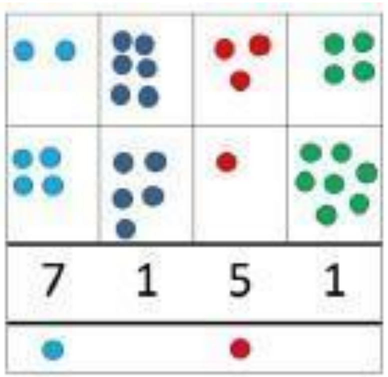
Progression in Calculations

Addition

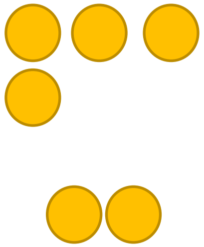
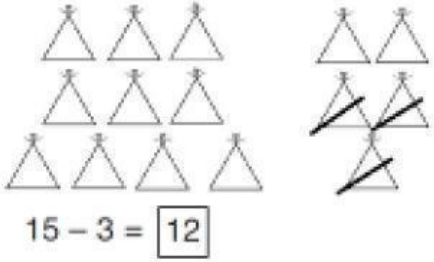
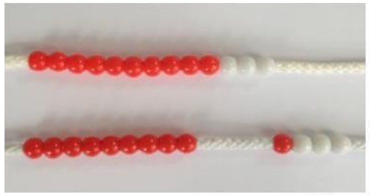
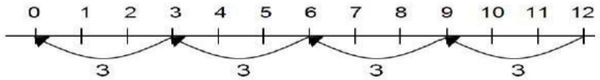
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the bigger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

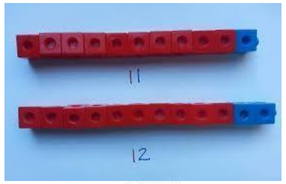
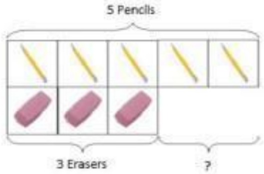
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

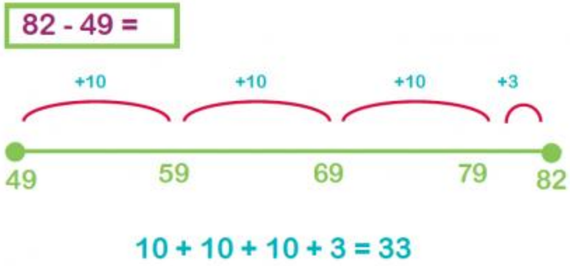
Column method- no regrouping	24 + 15 = 39 Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  	Laying out the numbers to be added in columns, and starting by adding the ones. <u>Calculations</u> 21 + 42 = $\begin{array}{r} 21 \\ +\ \underline{42} \end{array}$

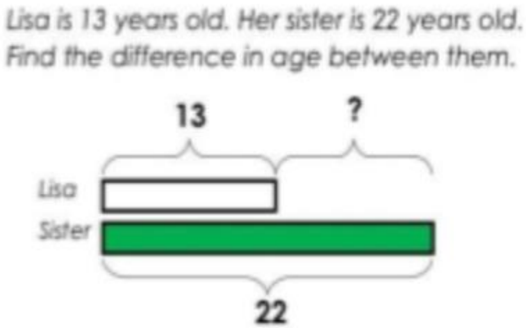
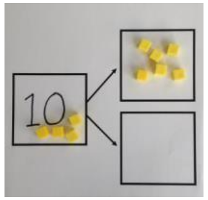
Column method-regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.  	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 536 + 85 621 1 1 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4  4 - 2 = 2	Cross out drawn objects to show what has been taken away.  	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the larger number and count back to the smaller number showing the jumps on the number line. This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

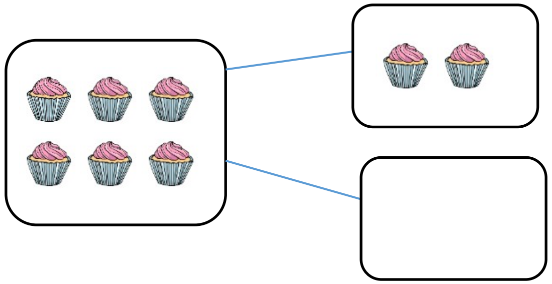
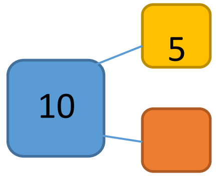
<table>
<tr>
<td>Find the difference</td>
<td>Compare amounts and objects to find the difference.

Use cubes to build towers or make bars to find the difference


Use basic bar models with items to find the difference</td>
<td>Count on to find the difference.


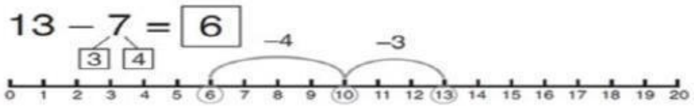
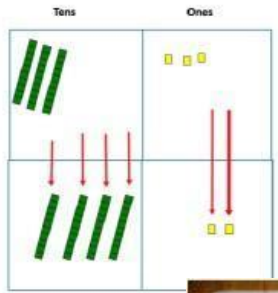
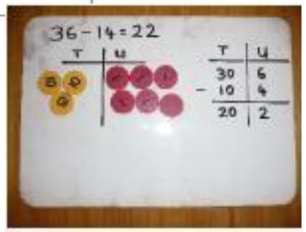
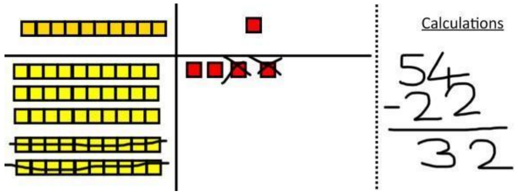
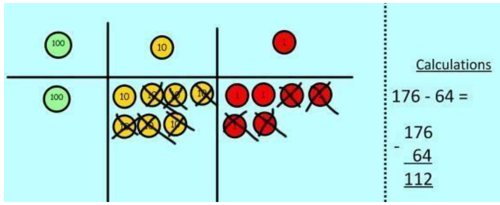
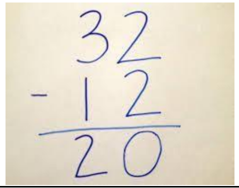
Draw bars to find the difference between two numbers.

</td>
<td>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</td>
</tr>
<tr>
<td>Part Whole Model</td>
<td>

Link to addition - use the part whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?

10 - 6 =</td>
<td>Use a pictorial representation of objects to show the part whole model.
</td>
<td>

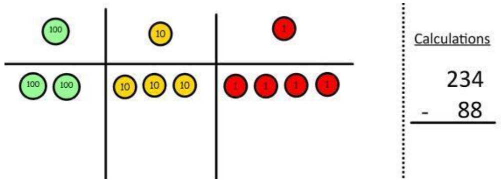
Move to using numbers within the part whole model.</td>
</tr>
</table>

Make 10	14 – 5 = 9  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. Children should count below the number line.	16 – 8 = 8 How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.  	  Draw the Base 10 or place value counters alongside the written calculation to help to show working.	47 – 24 = 23 40 + 7 – 20 + 4 20 + 3 This will lead to a clear written column subtraction. 

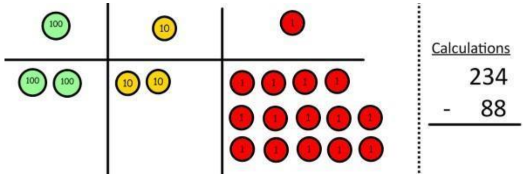
Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters

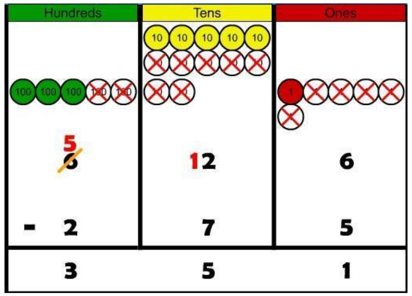


Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

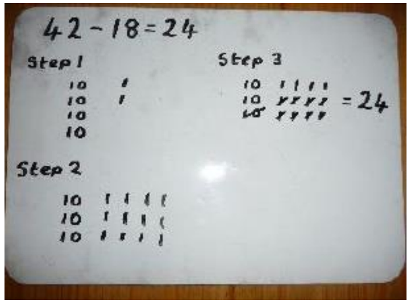


Now I can subtract my ones.

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

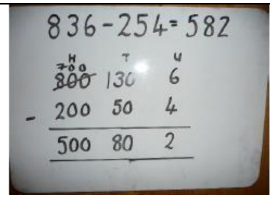


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

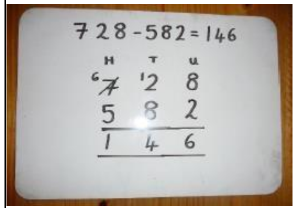


When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.



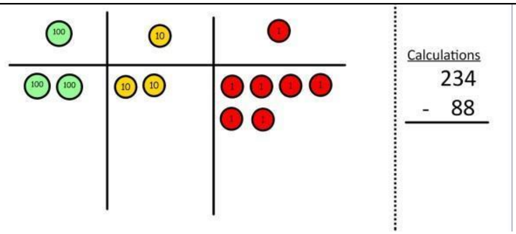
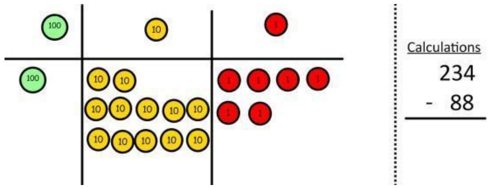
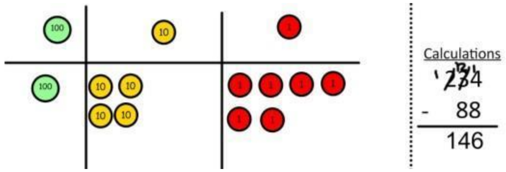
Children can start their formal written method by partitioning the number into clear place value columns.



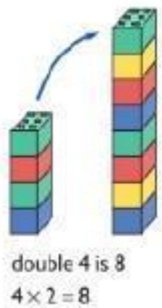
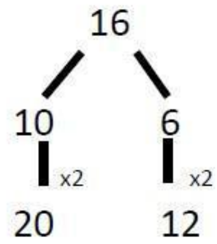
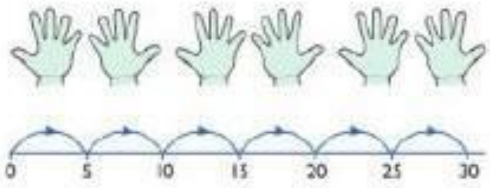
Moving forward the children use a more compact method.

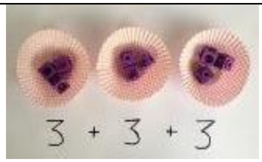
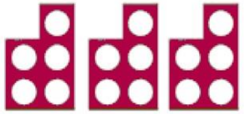
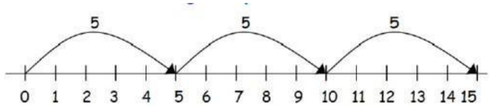
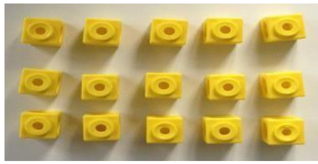
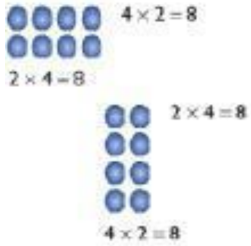
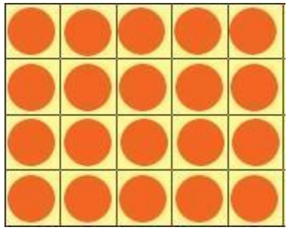
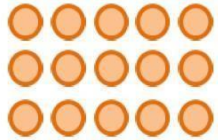
This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{cccccc} & & ^{5} & ^{12} & & ^{1} \\ & 2 & \cancel{6} & \cancel{3} & . & 0 \\ - & & 2 & 6 & . & 5 \\ \hline & 2 & 3 & 6 & . & 5 \end{array}$$

	  Now I can take away eight tens and complete my subtraction  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

Multiplication

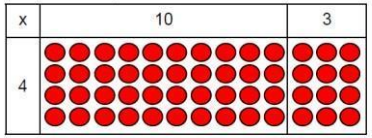
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number. Double 4 is 8 	Partition a number and then double each part before recombining it back together. 
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition	   Use different objects to add equal groups.	  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Grid Method			

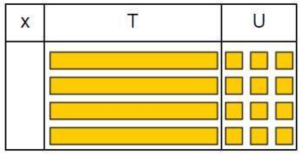
Grid Method

Show the link with arrays to first introduce the grid method.



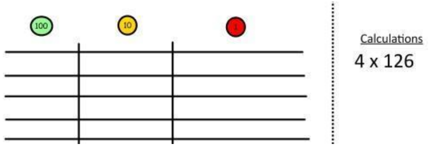
4 rows of 10 4 rows of 3

Move on to using Base 10 to move towards a more compact method.

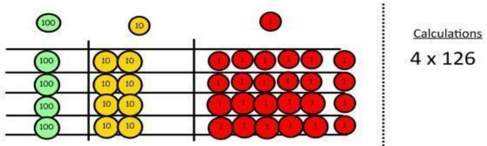


4 rows of 13

Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



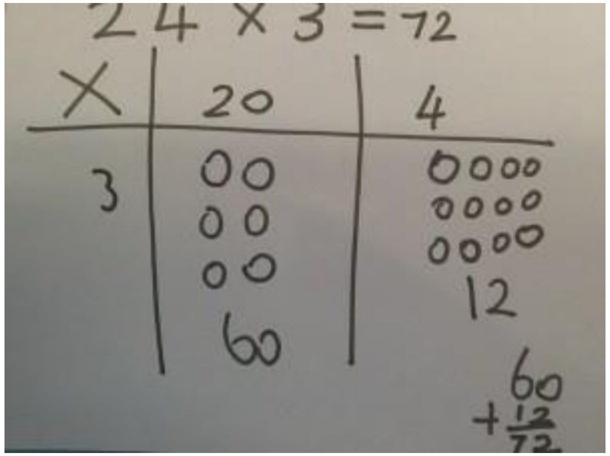
Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

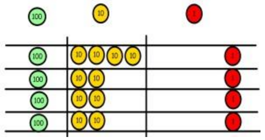
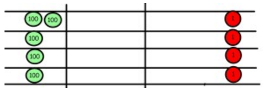
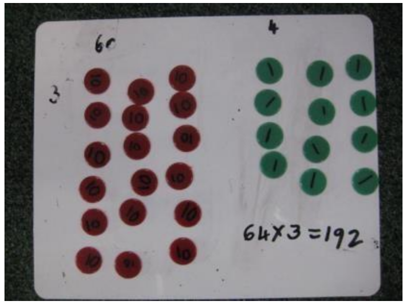
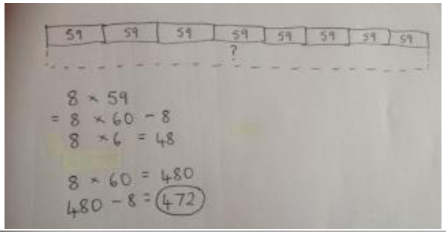
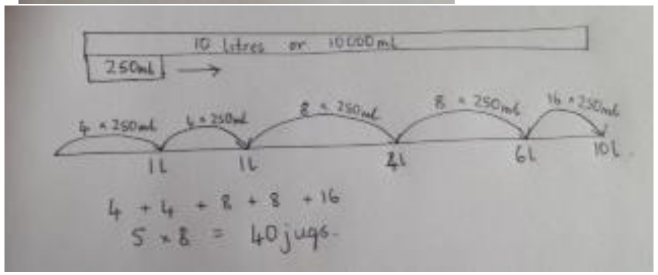
×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

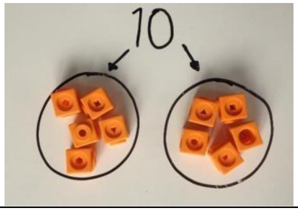
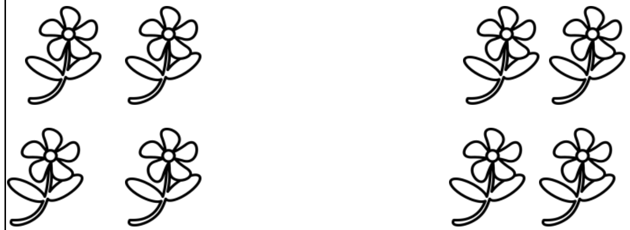
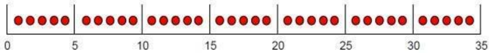
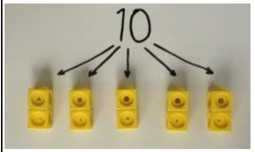
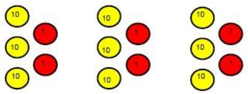
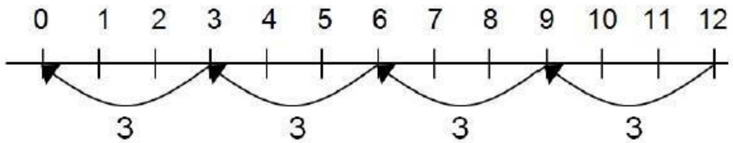
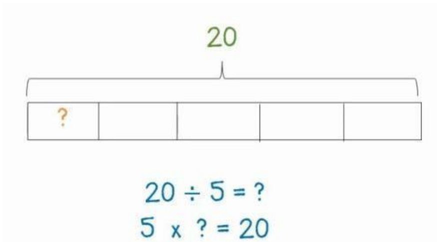
	10	8
10	100	80
3	30	24

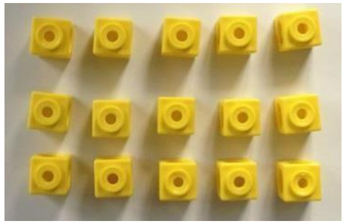
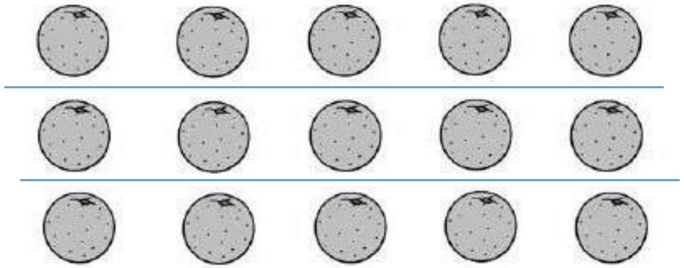
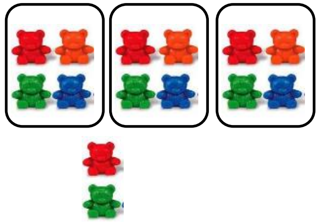
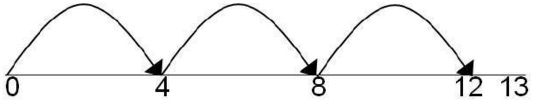
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

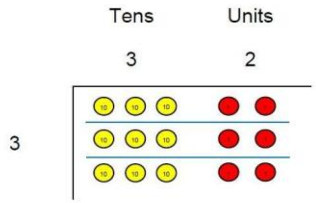
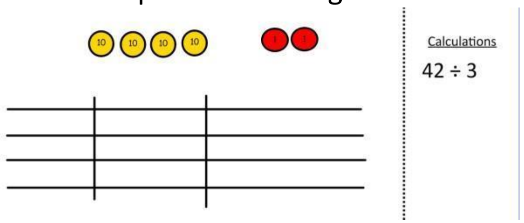
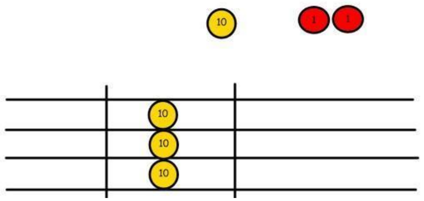
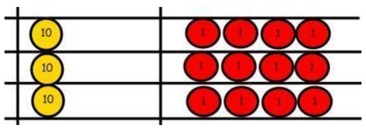
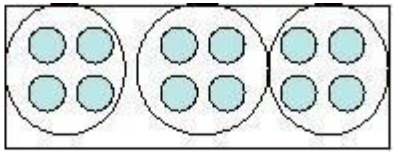
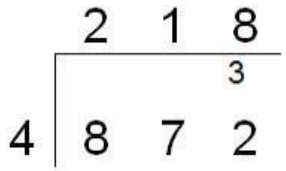
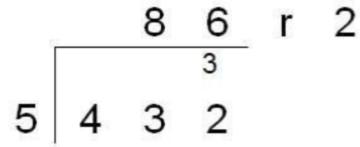
	  Then you have your answer.		
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768

			This moves to the more compact method. $\begin{array}{r} 74 \\ \times\ 63 \\ \hline 12 \\ 210 \\ 240 \\ +\ 4200 \\ \hline 4662 \end{array}$ $\begin{array}{r} 502 \\ \times\ 336 \\ \hline 3012 \\ 15060 \\ +\ 150600 \\ \hline 168672 \end{array}$

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $\boxed{8 \div 2 = 4}$	Share 9 buns between three people. $9 \div 3 = 3$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.    $96 \div 3 = 32$ 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = 4 r 2 Divide objects between groups and see how much is left over. 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 ↑ dividend ↑ divisor ↑ quotient ↑ remainder

Short division	 Use place value counters to divide using the bus stop method alongside.  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  Then we count how many in one group giving the answer 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.